$$2y^2(a^2+x^2) - 4ay(a^2-x^2) + (a^2-x^2)^2 = 0$$

$$y = \frac{a(a^2-x^2)}{a^2+x^2} \pm \frac{(a^2-x^2)^{3/2}}{\sqrt{2}(a^2+x^2)}$$

$$A = \int_0^a (y' - y'')\,dx = 2\int_0^a \frac{(a^2-x^2)^{3/2}}{\sqrt{2}(a^2+x^2)}\,dx$$ for area on the right

$$2A = 2\sqrt{2}\int_0^a \frac{(a^2-x^2)^{\frac{1}{2}}(a^2-x^2)}{a^2+x^2}\,dx$$

$$2A = 2\sqrt{2}\,a^2\int_0^a \frac{(a^2-x^2)^{\frac{1}{2}}}{a^2+x^2}\,dx - 2\sqrt{2}\int_0^a \frac{x^2(a^2-x^2)^{\frac{1}{2}}}{a^2+x^2}\,dx$$ $x^2 = a^2+x^2-a^2$

$$= 2\sqrt{2}a^2\int_0^a \frac{(a^2-x^2)^{\frac{1}{2}}}{a^2+x^2}\,dx - 2\sqrt{2}\int_0^a \frac{(a^2+x^2-a^2)(a^2-x^2)^{\frac{1}{2}}}{a^2+x^2}\,dx$$

$$= 4\sqrt{2}a^2\int_0^a \frac{(a^2-x^2)^{\frac{1}{2}}}{a^2+x^2}\,dx - 2\sqrt{2}\int_0^a (a^2-x^2)^{\frac{1}{2}}\,dx$$

$$-2\sqrt{2}\int_0^a (a^2-x^2)^{\frac{1}{2}}\,dx = -2\sqrt{2}\,\frac{\pi}{4}a^2 = -\frac{\pi a^2\sqrt{2}}{2}$$ = part of result

Now taking up the first term & leaving off for present the factor $4\sqrt{2}a^2$, let us find $\int_0^a \frac{(a^2-x^2)^{\frac{1}{2}}}{a^2+x^2}\,dx$

Assume $x = a\sin\varphi$, when $x = 0$, $\varphi = 0$ & when $x = a$, $\varphi = \frac{\pi}{2}$

$$\int_0^a \frac{(a^2-x^2)^{\frac{1}{2}}}{a^2+x^2}\,dx = \int_0^{\frac{\pi}{2}} \frac{1-\sin^2\varphi}{1+\sin^2\varphi} = \int_0^{\frac{\pi}{2}} \frac{d\varphi}{1+\sin^2\varphi} - \int_0^{\frac{\pi}{2}} \frac{\sin^2\varphi}{1+\sin^2\varphi}$$

$$= 2\int_0^{\frac{\pi}{2}} \frac{d\varphi}{1+\sin^2\varphi} - \int d\varphi = 2\int_0^{\frac{\pi}{2}} \frac{d\varphi}{1+\sin^2\varphi} - \frac{\pi}{2}$$

Restoring the coefficient to the last term, we have $-4\sqrt{2}a^2\frac{\pi}{2}$ for a second part of result.

Adding the 1st & 2nd parts we have $-5\sqrt{2}a^2\frac{\pi}{2}$.